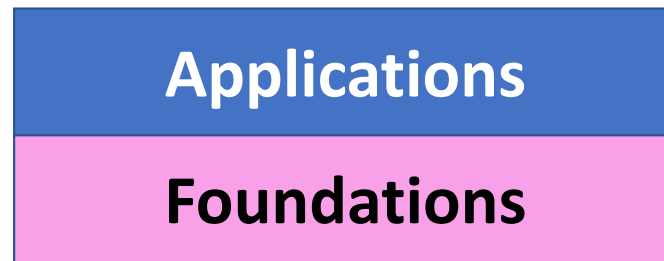


SE Foundations: Applications in the Innovation Ecosystem Context



For meeting of March 10, 2023

Contents

- Purpose and scope
- Context for applications / questions of interest
- History: What worked really well in the STEM revolution?
- Three observable phenomena
- Already known Phenomenon 1 theoretical foundations from the STEM revolution
- Where can STEM make better use of the STEM-based foundations?
- Are more foundations needed? What would they be about?

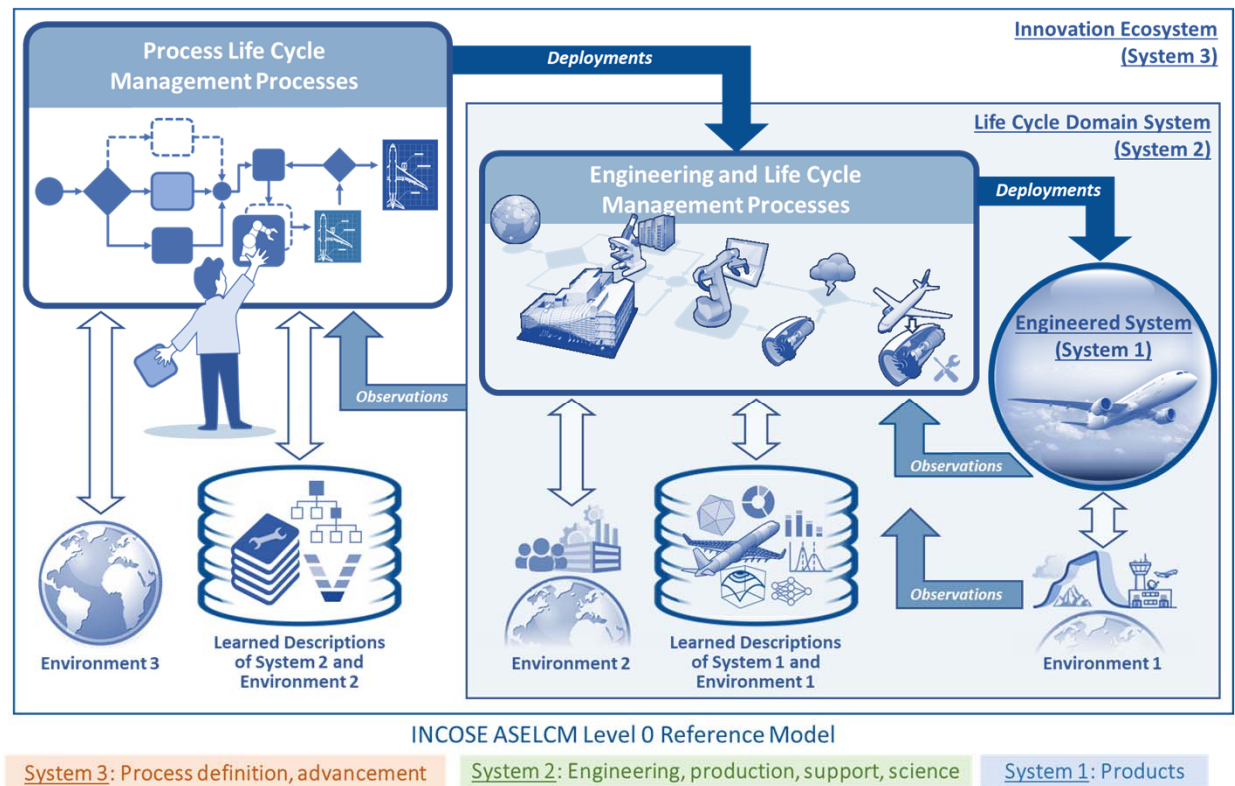
- Exploration of applications
- Discussion and next steps
- References

Purpose and scope

- The purpose of this material is to discuss specific example applications of theoretical foundations of systems science and engineering to improved understanding of innovation ecosystems.
- This includes identifying some of the questions such applications should address, and noting foundational elements that may help address those questions.
- Foundation elements noted include some long established in STEM (even if under-emphasized by the systems community) as well as needs and possibilities for additional foundation elements.
- This material is limited to a brief overview in support of collaboration by INCOSE working groups and the FuSE Foundations work stream, before, during and after the INCOSE IS2023.

Context for problems / applications of interest

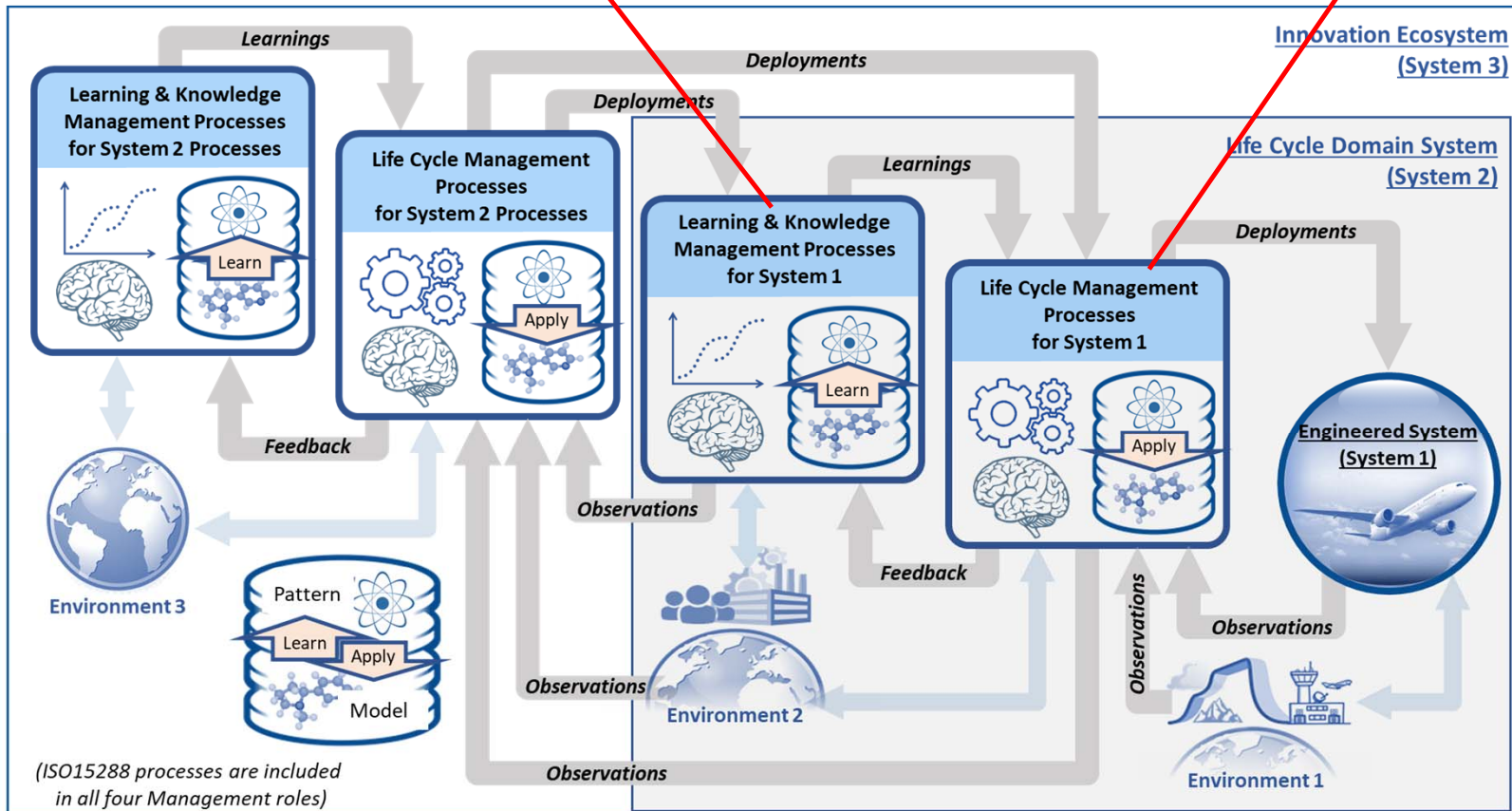
- The context of the problems of interest here can be well-described using the INCOSE Innovation Ecosystem Pattern to provide context--with a focus on its System 1 (Engineered System) and System 2 (System Life Management System).
- A framing of long historical interest to the INCOSE Patterns Working Group and others.
- It is also compatible with the context discussed by Prof. Oli de Weck at INCOSE IW2023.
- See the References.



Level 1 Decomposition

Responsible for learning about S1 and its environment.

Responsible for applying what is already known about S1 and its environment, to product S1 and manage its life cycle.

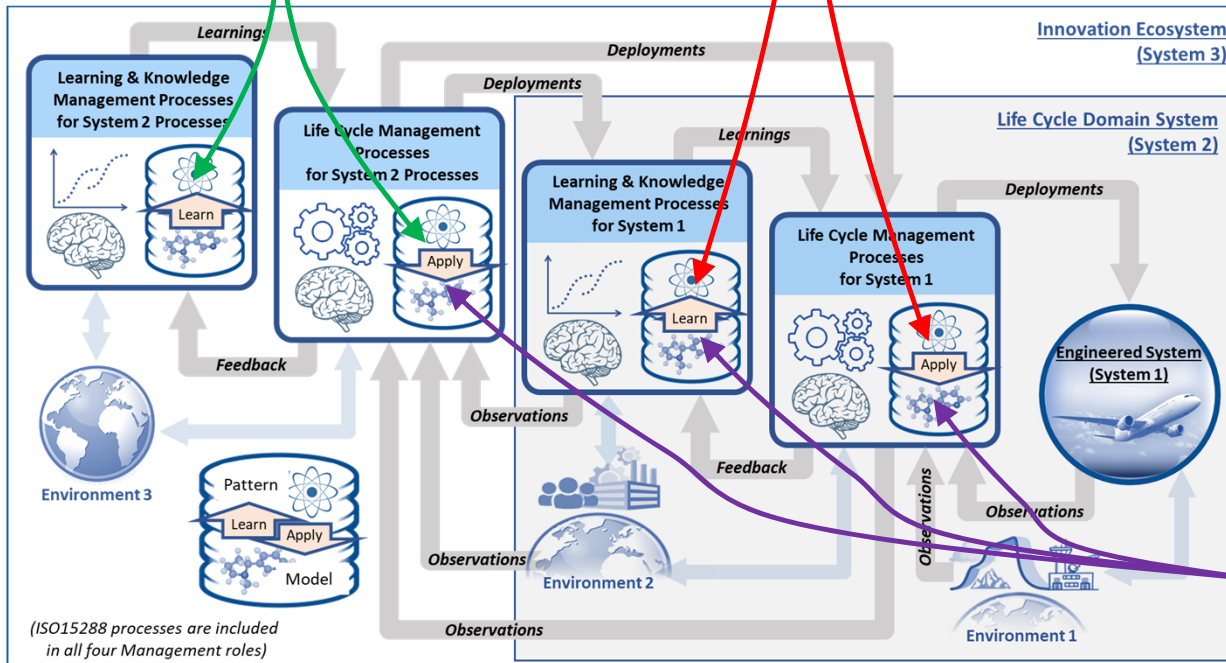


ASELCM Level 1 Reference Model

Question A: How to best represent System 1 when it includes data / cyber aspects? Socio-technical aspects?

Question B: Same as Question A, but for representing System 2.

Application questions of interest



ASELCM Level 1 Reference Model

Question C: Given effective representation, how to understand the time, cost, and uncertainty for an S2 program to deliver S1, as a function of (1) characteristics of S1 and its environment; (2) what S2 already knew about S1 and its environment before starting this program; and (3) the characteristics of S2 and its environment?

How S2 represents S1: History

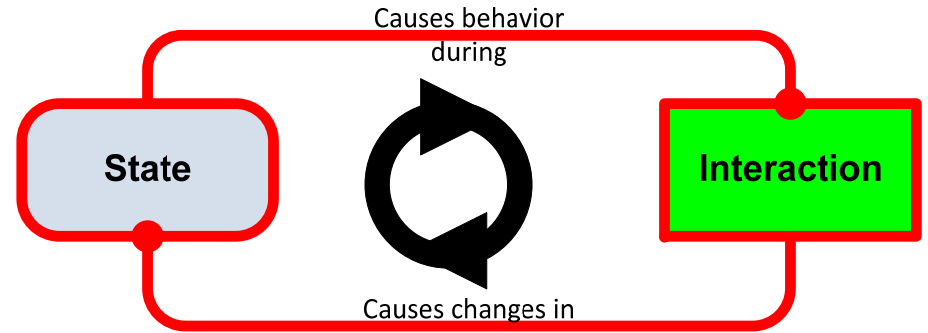
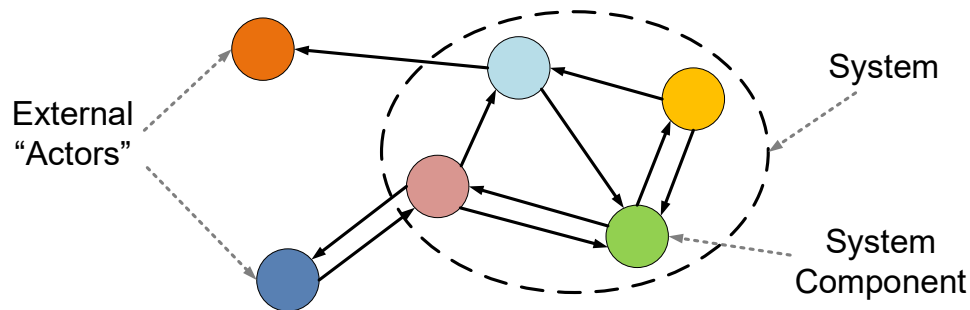
- 1700-2000: The STEM Revolution in mechanics, electrical science, chemistry, etc.:
 - Models emergent from physical sciences observation of phenomena.
 - Eventual application of those models to emerging engineering disciplines.
 - Striking unity of a few theoretical foundations across diverse types of phenomena was remarkable to the pioneers (e.g., M. Planck on Hamilton's Principle, etc.); the System Phenomenon.
 - This will be the focus of the "primer" joint project of our two working groups.
- 1950-2000: The emergence of contemporary systems engineering:
 - Driven by twentieth century needs to improve coordination of the work of the specific engineering disciplines and their connection to stakeholder goals.
 - Consequently, not supported or driven by phenomena-specific physical science in same ways as earlier engineering disciplines.
- More recent emergence of model-based methods for systems engineering:
 - But caught between model-based representation of information of the 1950-2000 SE flavor, versus phenomenological representation of S1 that is STEM-validated.
 - Some related challenges to representation of cyber and socio-technical systems.

What worked really well in STEM? What are the related STEM lessons about representing S1?

- The System Phenomenon (Phenomenon 1) is the context for the STEM core theoretical foundations of particular interest:
 - As the (Interaction) framework for classical and contemporary STEM that worked with such great impact for human life.
 - That impact will be described in the Primer project.
- Provided the theoretical framework for understanding the composition of larger phenomena from smaller ones:
 - And, larger engineered systems from smaller interacting parts.
 - That framework is not specific to only some disciplines—it is generic.

Formalizing System Terms and Representations

- Definition: *In the perspective described here**, by “System” we mean a collection of interacting system components:

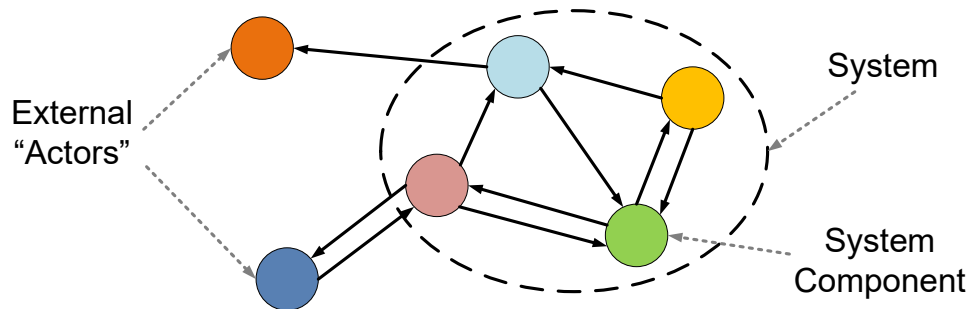


- By “interacting” we mean the exchange of energy, force, material, or information (all of these are “input-outputs”) between system components, . . .
- . . . through which one component impacts the state of another component.
- By “state” we mean a property of a component that impacts its input-output behavior during interactions. (Note the circular cause-effect definition chain here.)
- So, a component’s “behavior model” describes input-output-state relationships during interaction—*there is no “naked behavior” in the absence of interaction.*
- The behavior of a system involves emergent *states of the system as a whole*, exhibited in its behavior during its own external interactions, resulting in observable holistic aspects.

(* Other world view definitions of “System” are acknowledged; there are reasons for our minimalist choice of definitions.)

The System Phenomenon

- Phenomena of the hard sciences in all instances occur in the presence of special cases of the (generalized) “System Phenomenon”:
 - *The System Phenomenon: System behavior emerges from the interaction of behaviors (phenomena themselves) of system components a level of decomposition lower.*
- Each emergent phenomenon is visible through the interaction-based behavior of the larger system with its own external environment:



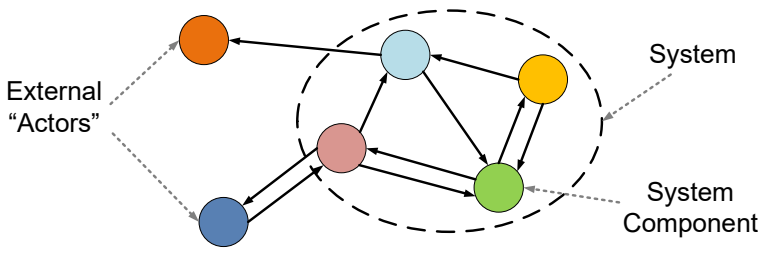
The combinatorial nature of emergent phenomena can be unpredictably diverse, as well as unlike the component behaviors.

- The resulting “patterns” of recurring larger-scale behavior become the basis for recognition, mathematical laws of motion or other hard science, heuristics, rules of thumb, intuition, prediction, or other exploitation of those regularities.
- Phenomena in the “softer” domains in all instances likewise occur in the presence of cases of the above System Phenomenon, even though the domain-specific phenomena, input-outputs, states, and behaviors are different.

STEM Triumphed for Large Subsets of the System Phenomenon

Engineering Discipline	Phenomena Special Case	Scientific Basis	Scientific Laws
Mechanical Engineering	Mechanical Phenomena	Physics, Mechanics, Mathematics	Newton's Laws
Chemical Engineering	Chemical Phenomena	Chemistry, Mathematics. . . .	Periodic Table
Electrical Engineering	Electromagnetic Phenomena	Electromagnetic Theory	Maxwell's Equations
Civil Engineering	Structural Phenomena	Materials Science, . . .	Hooke's Law, etc.
Semiconductor Eng'g	Semiconductor Phenomena	Solid State Physics, . . .	Quantum Mechanics

- For each such emergent phenomenon¹, the emergent interaction-based behavior of the larger system is a stationary state space trajectory of the action integral:



$$S = \int_{t_1}^{t_2} L(x, \dot{x}, t) dt ; \delta[S] = 0 \quad \leftarrow \text{(Hamilton's Principle}^1)$$

- Reduced to simplest forms, the resulting equations of motion (or if not solvable, simulated/observed paths) provide “physical laws” subject to scientific verification—an amazing foundation supporting all above phenomena.

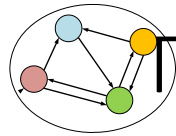
(1) When stated with rigor, special cases for non-holonomic constraints, irreversible dynamics, discrete systems, data systems, etc., led to alternatives to the variational Hamilton's Principle—but the interaction-based structure of the System Phenomenon remained, and the underlying related Action and Symmetry principles became the basis of modern theoretical physics.

The above generalization is long known:
Max Planck on Hamilton's Principle
(aka Principle of Least Action)



*“It [science] has as its highest principle and most coveted aim the solution of the problem to condense all natural phenomena which have been observed and are still to be observed into one simple principle, that allows the computation of past and more especially of future processes from present ones. ...Amid the more or less general laws which mark the achievements of physical science during the course of the last centuries, the **principle of least action** is perhaps that which, as regards form and content, may claim to come nearest to that ideal final aim of theoretical research.”*

Max Planck, as quoted by Morris Kline, *Mathematics and the Physical World* (1959) Ch. 25:
From Calculus to Cosmic Planning, pp. 441-442



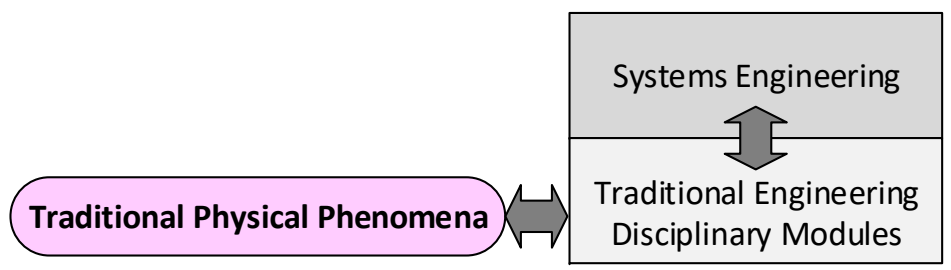
The System Phenomenon: Conclusion

- Each of the so-called “fundamental” phenomena-based laws’ mathematical expression (Newton, Maxwell, Schrodinger, et al) is derivable from the above formulation—as shown in many discipline-specific textbooks.
- So, instead of Systems Engineering lacking the kind of theoretical foundation the “hard sciences” bring to other engineering disciplines, . . .
 - It turns out that all those other engineering disciplines’ foundations are themselves dependent upon the System Phenomenon and Hamilton’s Principle mathematical expression of the inductive pattern from Level N to Level N+1 (others followed with generalizations and extensions).
 - **SO**, the underlying math and science of systems provides the theoretical basis already used by all the hard sciences and their respective engineering disciplines.
 - It is not Systems Engineering that lacks its own foundation—instead, it has been providing the so-called foundations claimed by each of the other disciplines!
 - This opens a new perspective on how Systems Engineering and Systems Science can relate to the other, better-known disciplines, as well as future domains . . .

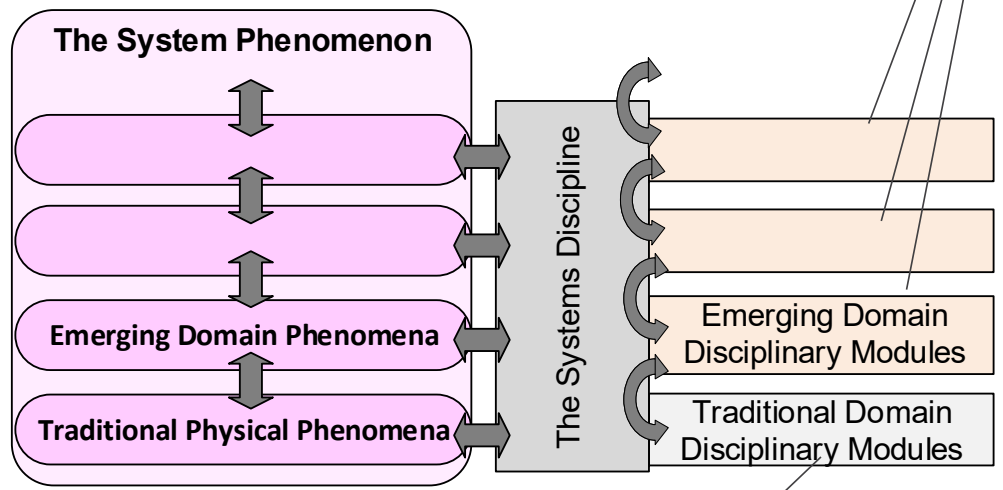
- The System Phenomenon and its supporting mathematics (Hamilton et al) provide the inductive ladder, explaining (*) theory of each new level in terms of the previous level.
- As higher-level system patterns are discovered, represented, validated, taught, and practiced, they become “emergent domain disciplinary frameworks”.
- This is evident in the history of scientific and engineering domains and disciplines, and newer emerging ones.

- | | |
|--------|--|
| Future | <ul style="list-style-type: none"> • Distribution networks • Biological organisms, ecologies • Market systems and economies • Health care delivery • Systems of conflict • Systems of innovation |
| Recent | <ul style="list-style-type: none"> • Ground Vehicles • Aircraft • Marine Vessels • Biological Regulatory Networks |

Traditional view:

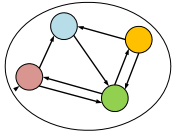


Future view:



* Explaining after their discovery, but generally not predicting them before. See P. W. Anderson, Att. 1.

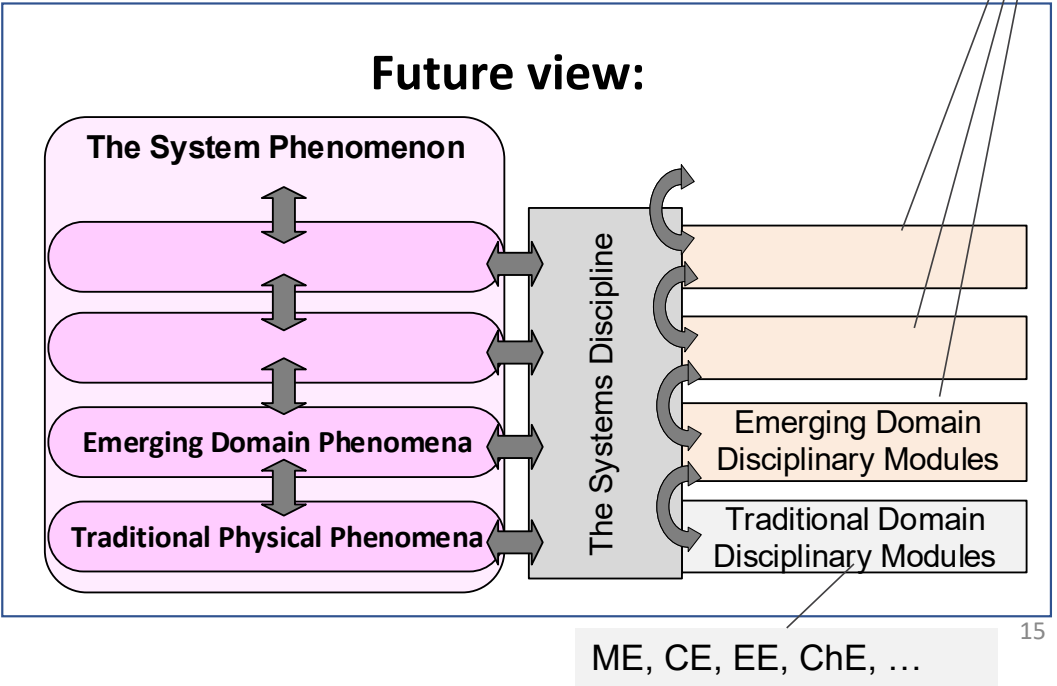
ME, CE, EE, ChE, ...



Impacts on Semantic Structure Emerge Uniquely for Each Emergent Domain

- | | |
|--------|--|
| Future | <ul style="list-style-type: none"> • Distribution networks • Biological organisms, ecologies • Market systems and economies • Health care delivery • Systems of conflict • Systems of innovation |
| Recent | <ul style="list-style-type: none"> • Ground Vehicles • Aircraft • Marine Vessels • Biological Regulatory Networks |

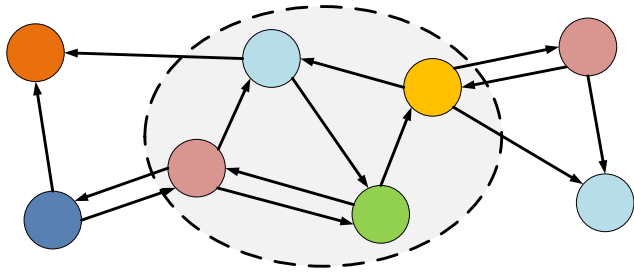
- New interactions (e.g., on the Internet) lead to new domains—each with new structure, new named things (roles), attributes, and relationships.
- Each new domain arising from new interactions thus creates a new ontology (domain specific language).
- So, a single “master ontology” is thus never enough!
- Domain ontologies are about semantic structure, not about quantitative mathematical aspects.
- Human skills and tools for language and meaning are called into play—different than quantitative skills and tools. Calls upon System Thinking.
- The related ontology frameworks thus have both structural semantic and quantitative math aspects.
- Here designers face a different “reverse” problem than scientists: Seeking to discover structure to produce interaction behaviors to deliver benefits.



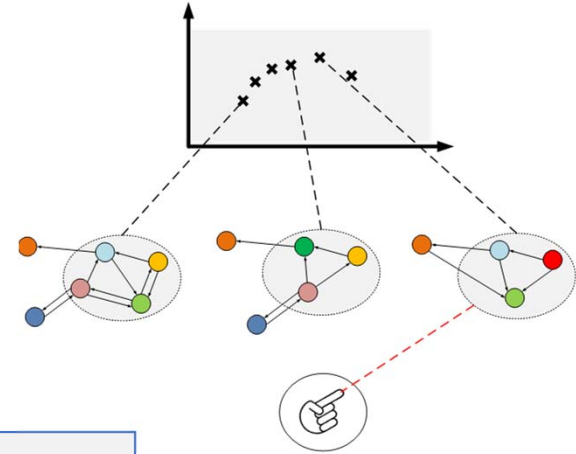
Three observable phenomena

- The following three observable phenomena are from our 2020 input to INCOSE Vision 2034 theoretical foundations section.
- Our initial interest is Phenomenon 1: The System Phenomenon
 - Especially relevant for Questions A and B.
- Then we will come back to Phenomenon 2 and 3 later below:
 - Potentially relevant for Question C.

1. The System Phenomenon

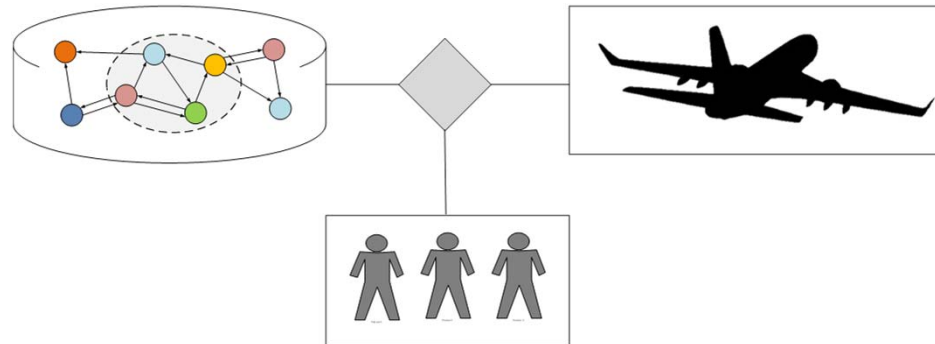


2. The Value Selection Phenomenon



Three Foundational Systems Phenomena

3. The Model Trust by Groups Phenomenon



Already known Phenomenon 1 theoretical foundations
from the STEM revolution



- **Hamilton's Principle**: Was already strong enough to generate all the fundamental phenomena of physics, from Newton through Feynman. *But later followed by Hamilton's further insight beyond least action.*

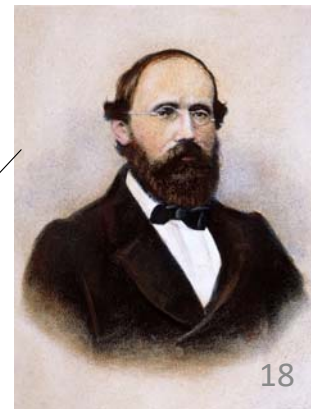


- **Noether's Theorem**: Deeper insight into the connection of Hamilton's principle to Symmetry and Conservation Laws. Both symmetries and patterns have fixed (invariant) and variable parts.



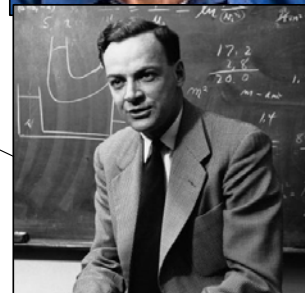
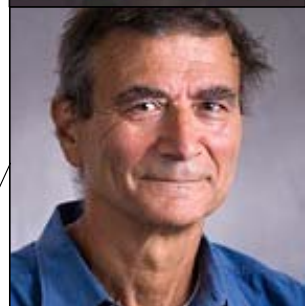
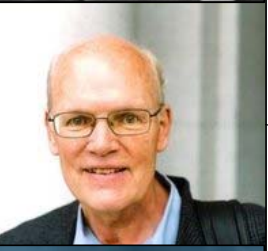
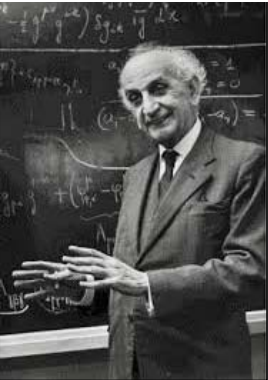
- **D'Lambert's Principle**: Older than Hamilton, but wider in scope than Hamilton's Principle, adding non-holonomic constraints, dissipative systems

- **Bernhard Riemann**: Embedded Manifold spaces further generalize representation of complex dynamics.



Already known Phenomenon 1 theoretical foundations
from the STEM revolution

- **Cornelius Lanczos**: Master elucidator of Analytical Mechanics
- **Prigogine, Sieniutycz, Farkas**: Irreversible and large scale thermodynamic systems
- **JE Marsden, A Bloch, Marston Morse**: Non-Holonomic Control Systems, Discrete Mechanics; Symbolic Dynamics, Discrete Hamilton's Principle; Discrete Noether's Theorem
- **Ed Fredkin, Charles Bennett, Tomas Toffoli, Richard Feynman**: Information Systems and Automata



A STEM-based Phenomenon 1 Mechanics for Generic Systems:

- Newton (1643-1727) and others mostly considered what we would today consider mechanical systems, but effectively introduced a more generic ontological framework of evolving States, Interactions, and Forces (among other things), and the calculus (and differential equations) for describing that framework mathematically, including its expression of Newton's Laws, in 3-D Euclidean space.
- Lagrange (1736-1813) and others advanced the foundational framework by addition of kinetic and potential energy, variation of time evolutions of state trajectories, and mathematical relationships expressed by the Lagrangian L and its Action S integral, leading to the principle of stationary action, the Euler-Lagrange Equation, and what came to be called Hamilton's Principle, in configuration coordinate (X_i, \dot{X}_i) space.
- Hamilton (1805-1865) and others further advanced the framework by (Legendre) transforming the Lagrangian L (and its configuration coordinates space) to the Hamiltonian H (and its canonical $\{X_i, P_i\}$ coordinate space). This included introduction of generalized momentum (p_i) defined (without dependence on mechanical phenomena) as $p_i = \frac{\partial H}{\partial \dot{X}_i}$. This opened the framework to rigorously expressed generalized momenta and energy for cyber, social, and other systems. Hamilton's Equations also opened the framework to direct integration methods so critical to digital simulation.
- Marsden (1942-2010) and others advanced the modern foundation of mechanics, including its framing using differential geometry and tangent bundles on manifolds.

A STEM-based Phenom 1 Mechanics for Generic Systems: Example thought experiment with Hamiltonians

- Hamilton got to the Hamiltonian H through a Legendre transform of the Lagrangian L, but we will just use a thought experiment here . . .
- Start by considering a continuous, deterministic, system. (STEM pioneers later generalized to discrete and probabilistic systems, but we'll start where they did.)
- The deterministic system's behavior over time is described by a generator function g that is unknown but represents determinism:

$$\frac{d\vec{x}}{dt} = \vec{g}(\vec{x}, t) \quad , \quad \text{where } \vec{x} = (x_1, x_2, \dots, x_n) \text{ is a list of state variables representing the state of the system at time } t.$$

- Note that *no assumption is made here about the nature of the system and its states, which may be mechanical, chemical, electrical, information, socio-technical, or otherwise.* The only assumptions are that there are states $\vec{x}(t)$, and a state trajectory “generator” g.
- By observing the system's behavior over time, we can collect $(\vec{x}, \dot{\vec{x}})$ pairs that implicitly provide information about an as yet unknown scalar function H (the Hamiltonian) that is a function of \vec{x} and \vec{p} . Here p represents “generalized momentum”, and is defined in terms of the H we are looking for. We seek an H that satisfies Hamilton's Equations:

$$\text{Hamilton's Equations: } \dot{x}_i = \frac{\partial H}{\partial p_i} \quad \dot{p}_i = -\frac{\partial H}{\partial x_i} \quad , \quad \text{where generalized momentum is defined by: } \vec{P} = \left\{ \frac{\partial H}{\partial \dot{x}_i} \right\}$$

- So, even though we don't have direct observations of P or H, we can imagine the problem of “find an H and a P such that the observed $(\vec{x}, \dot{\vec{x}})$ pairs are consistent with H and P.”
- It turns out that if you wanted to do that for real numerical observations, you can discover the numerical representation of H by using a learning machine algorithm on the observation data to “learn” the Hamiltonian! (See References.)
- So, for numerically observed $(\vec{x}, \dot{\vec{x}})$ pairs for any system (including cyber / data systems, social systems, System 2 of ASELCM Pattern, etc.), we can effectively generate that system's Hamiltonian (generalized total energy) and generalized momentum.

Predicting system behavior for any of those kinds of systems

- Once we have a Hamiltonian, we can use Hamilton's equations

$$\dot{x}_i = \frac{\partial H}{\partial p_i} \quad \dot{p}_i = -\frac{\partial H}{\partial x_i}$$

to predict the future trajectory of the system in canonical coordinates $(\overrightarrow{x(t)}, \overrightarrow{p(t)})$ —say, by numerical integration using symplectic integration software to integrate over time.

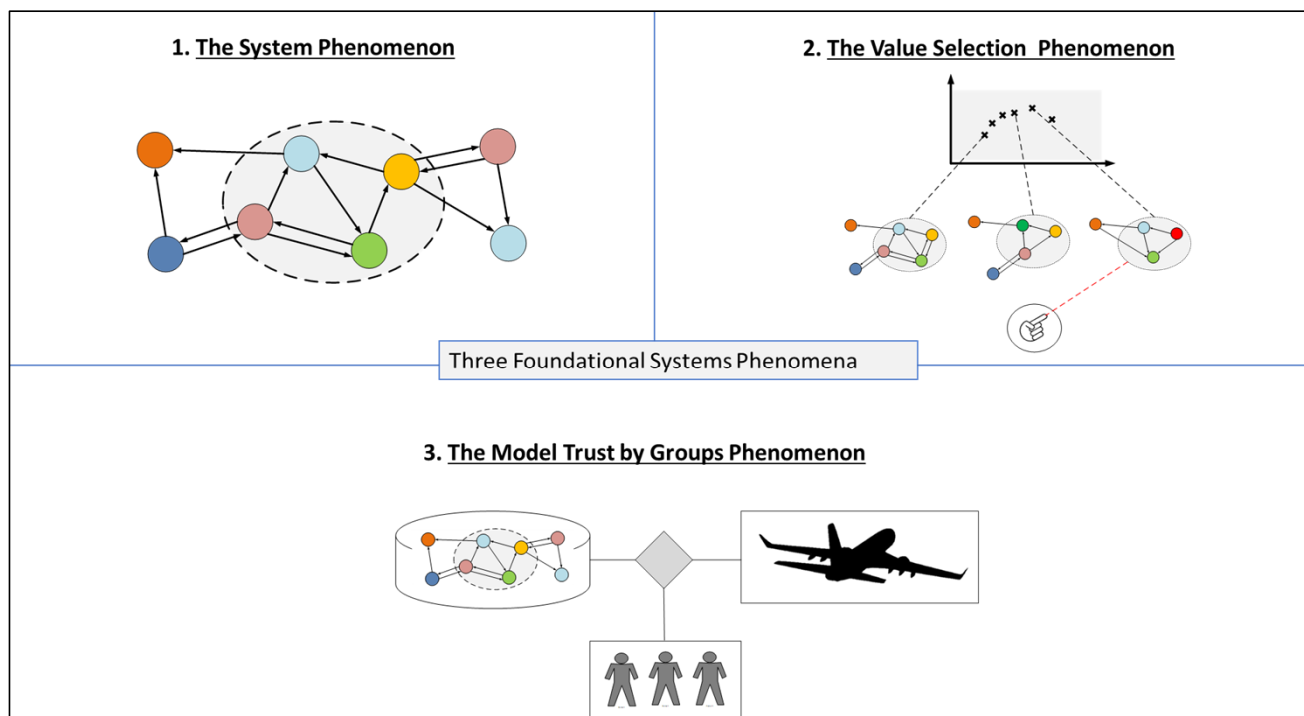
- Of course, if we actually obtained H by means of observing a real system and using the machine learning method described earlier above, then we don't have complete assurance that future behavior of the real system would necessarily follow the same Hamiltonian.

Where can SE make better use of the STEM ideas that already worked very well?

- Key concepts in STEM that worked with great impact:
 - Energy: Total, Potential (energy of state), Kinetic (energy of change). See the Hamiltonian.
 - Momentum: Tendency of a changing system state to keep changing. See Hamilton's definition of generalized momentum.
 - Force: See the derivative of generalized energy with respect to state and momentum.
 - These allow us to frame generic systems (including cyber and social systems) using already known ontology and mathematics from STEM.
- Domains where we'd like to apply those concepts directly:
 - Data, cyber-physical systems: Information states can be directly treated this way.
 - Socio-technical systems—for example, ASELCM System 2, the system of engineering and life cycle management
 - This can help with Question B, as well as Question A. It is also partial (table stakes) help with Question C.

Are more foundations needed? What would they be about?

- The above argues for use of existing and rather “classical” STEM theoretical foundations. Are there application areas where more foundation is needed?
 - Phenomenon 2 and Phenomenon 3 are areas in which further progress was already made during the twentieth century.

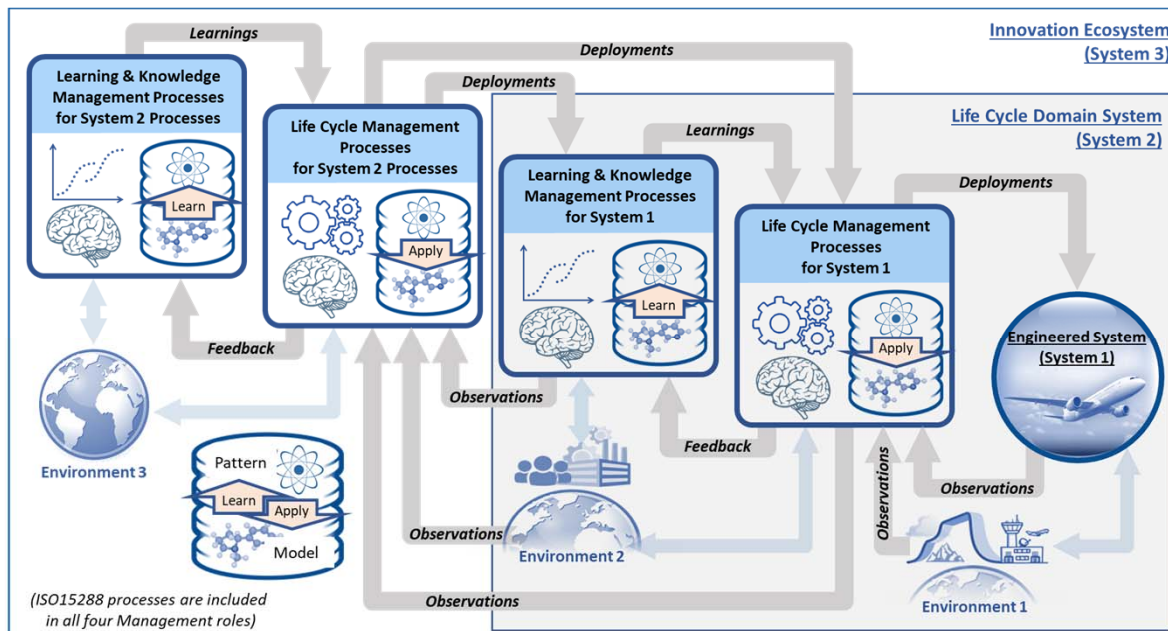


Example progress in support of better understanding of Phenomena 2 and 3:

- Ernst Ising (1900-1998) worked on a problem assigned by Wilhelm Lenz (1888-1957) that became known in statistical mechanics as the Ising Model of networks of interacting nodes, extended many times by others since then.
- John Hopfield (1933-) and others advanced theoretical models of neural networks that include network energy-based dynamics of learning.
- Rudolph Kalman (1930-2016) developed optimal estimation theory (Kalman-Bucy Filter) of the theorem of Thomas Bayes (1702-1762) to produce estimates in probabilistic environments that include predictions of propagation of Bayesian uncertainties for dynamical systems widely used in navigation and communication among other areas.
- Daniel Kahneman (1934-) advanced understanding of behavioral economics and related performance of human decision-making about perceived value and selection.
- All the above contributions advance foundational support for Question C. ²⁵

System 2: Engineering as a learning process

- We are interested in the Engineering portion of System 2:
 - Engineering processes consume and produce information.
 - Over the engineering cycle, this may be seen as managing (reconciling) a well-known set of inconsistencies, until sufficient consistency is achieved.
 - This is in effect a learning process, in which S2 learns about S1 and its environment.



ASELCM Level 1 Reference Model

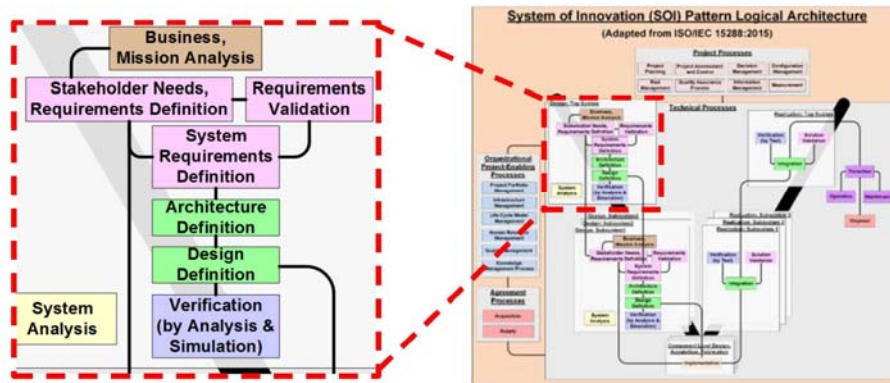
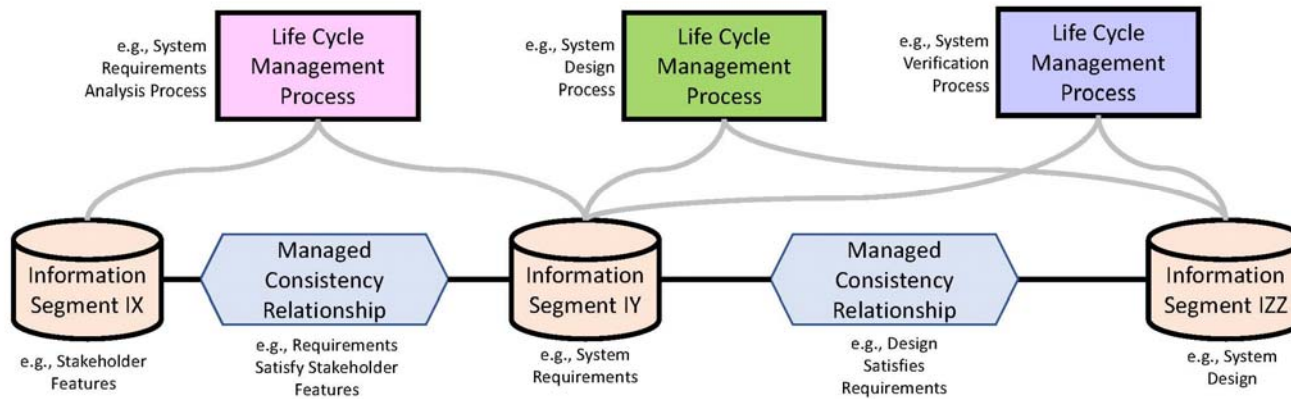
2023 Annual INCOSE International Workshop
 HYBRID EVENT
 Torrance, CA, USA
 January 28 - 31, 2023

**Discussion and References --
 Decision Analysis Patterns**

From: INCOSE MBSE Patterns Working Group
 For: INCOSE Decision Analysis Working Group
 Meeting of Jan 30, 2023

V1.5.5 Copyright © 2023 by W. D. Schindel. Permission granted to INCOSE to publish and use www.incose.org/IW2023

A few consistency examples in S2 context:



15

More than a learning process:

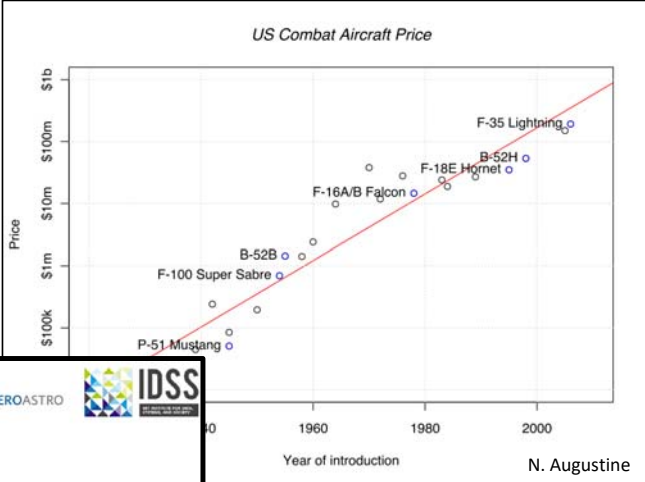
- S2 as a system drives toward minimizing the size of the inconsistencies over time.
- This can also be viewed as a set of gap energies that are being minimized by learning and selecting information configurations that minimize those energies (gaps).
- As in the case of Hopfield Nets, learning can be viewed as an energy minimization process.
- There are also steps in which selections are made from among alternatives.
- So, this is also viewed as a case of value selection, with selection forces in play.
- Relevant Phenom 1 “state variables” for Questions B and C include size of inconsistency gaps as well as expenditures of resources, time, uncertainties, etc.

There are also uncertainties involved concerning that information, and System 2 group phenomena

- The S2 learning process may be viewed as iteratively creating a model that is sufficiently consistent with certain external and internal aspects.
- But, there can also be variation and noise involved, introducing issues of uncertainty and trust.
- This is nevertheless a learning process, set in a context of uncertainty.
- So, it can be viewed as gaining confidence (trust) in a model.
- But, there is not just one “truster” involved: there are group dynamics involved in propagation of trust and other information across groups.

Historical and recent Question C efforts

- Prof. Oli de Weck et al, leading up to IW2023 FuSE talk
- Historical industry efforts: COSYSMO (Valerdi, Boehm, et al)



COSYSMO: A Systems Engineering Cost Model

Barry W. Boehm, Donald J. Reifer, Ricardo Valerdi
University of Southern California – Center for Software Engineering
941 W. 37th Place, SAL Room 328
Los Angeles, CA 90089-0781
(213) 740-8163
boehm@sunset.usc.edu, dreifer@earthlink.net, rvalerdi@sunset.usc.edu

Abstract

Building on the synergy between Systems Engineering and Software Engineering, the Center for Software Engineering (CSE) at the University of Southern California (USC), has initiated an effort to develop a parametric model to estimate Systems Engineering costs. The goal of this model, called COSYSMO (Constructive Systems Engineering Cost Model), is to more accurately estimate the costs of systems life cycle. Similarly, large projects cannot succeed without applying good Systems Engineering practices and principles in such a way that unify systems and software engineering efforts [Boehm 1994]. The recent release of the Capability Maturity Model Integration v1.1 (CMMI®) represents one such effort that is aimed towards integrating Systems and Software Engineering processes. The advantages of information the most disciplines are clear, but little

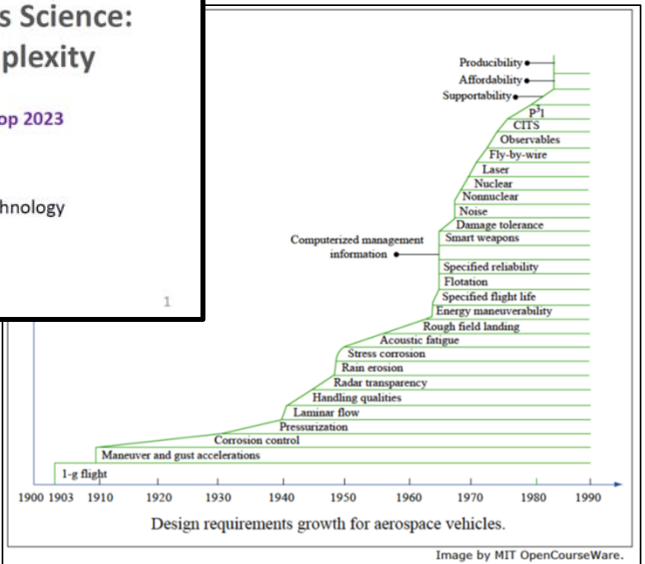
MIT Massachusetts Institute of Technology AEROASTRO IDSS

The First Law of Systems Science: Conservation of Complexity

INCOSE International Workshop 2023

Prof. Olivier de Weck
Massachusetts Institute of Technology
deweck@mit.edu

1



Lessons Learned From Industrial Validation of COSYSMO

17th INCOSE Symposium

Dr. Ricardo Valerdi
Garrv Roedler
Dr. Gan Wang

Marilee Wheaton
THE AEROSPACE CORPORATION
Dr. John Rieff
Raytheon
BAE SYSTEMS

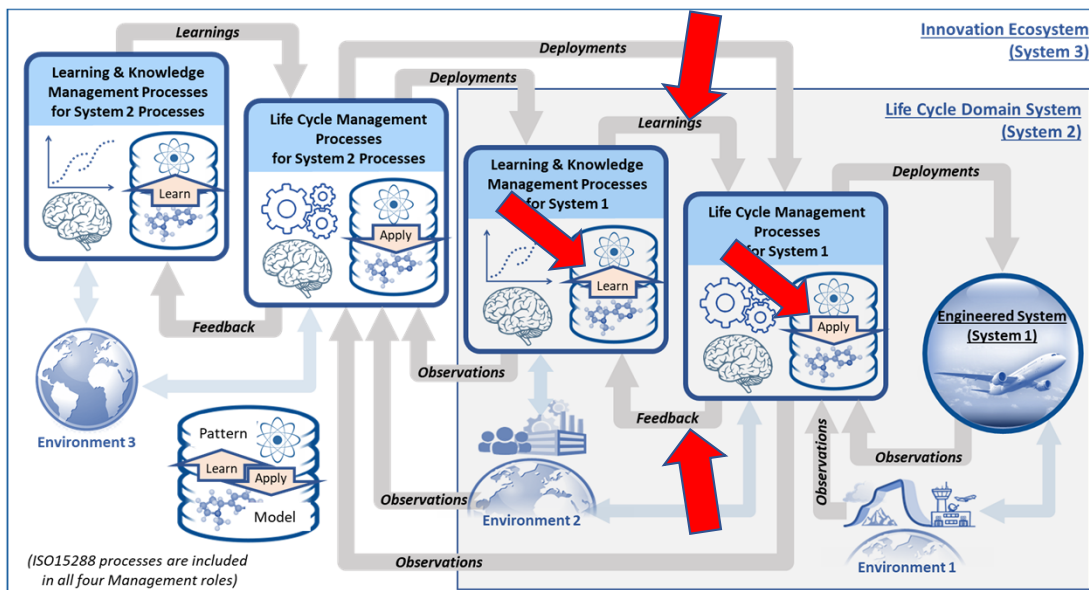
LOCKHEED MARTIN

SEA

INCOSE 2007

Use of incremental learning: A central issue

- The ASELCM Pattern, from the Patterns Learning Working Group, is especially concerned with the capture and application of learning across the organization.
- Learning recurrent patterns, in many forms, is central to this understanding.
- Question C was accordingly worded to reflect both what was known before a program starts, versus learned during the program.



ASELCM Level 1 Reference Model

Question C: Given effective representation, how to understand the time, cost, and uncertainty for an S2 program to deliver S1, as a function of (1) characteristics of S1 and its environment; (2) what S2 already knew about S1 and its environment before starting this program; and (3) the characteristics of S2 and its environment?

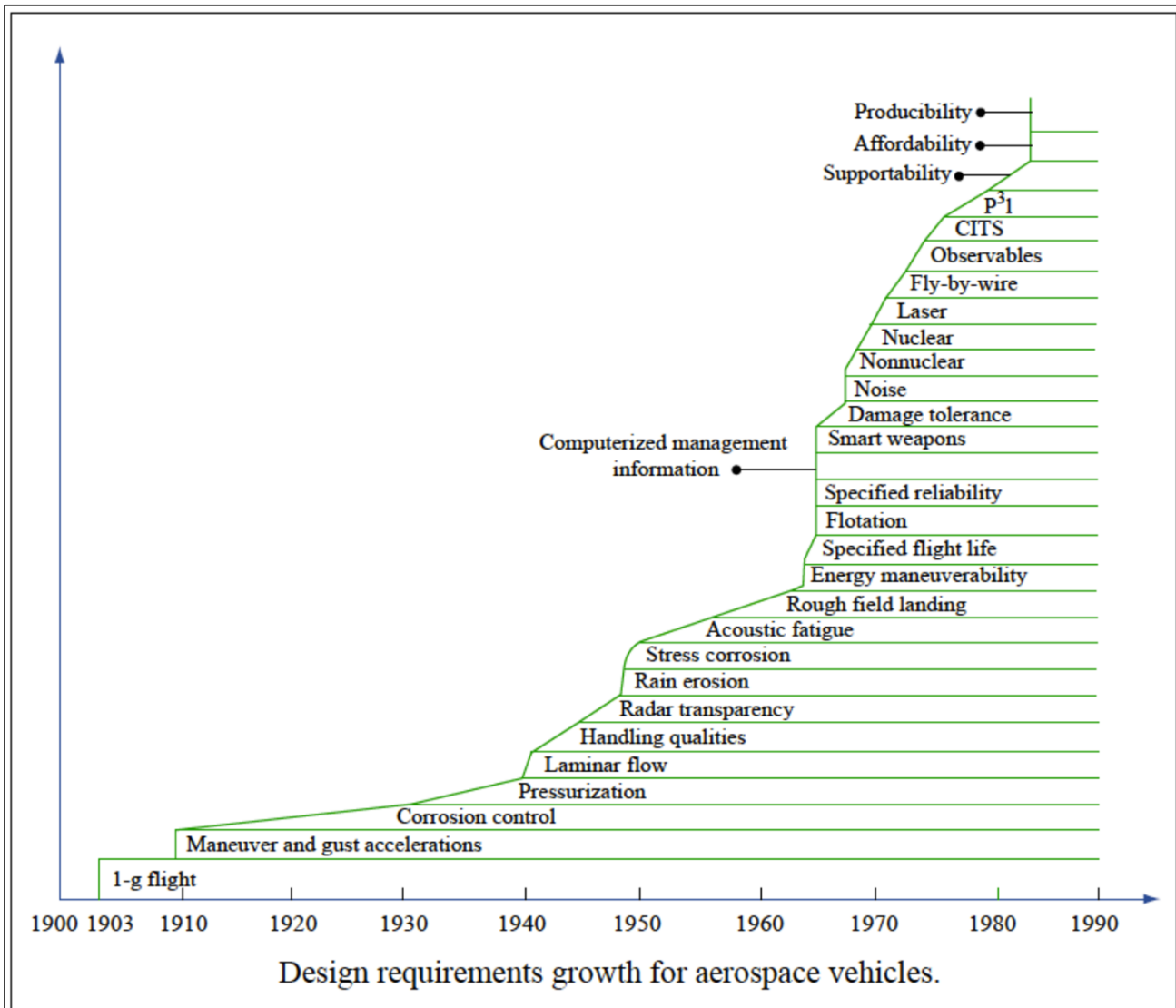


Image by MIT OpenCourseWare.

- These historical aircraft programs did not each start entirely from scratch for each program.
- The high complexity programs on the right did not start from the same place as the early programs on the left.
- So, the complexity of the S1 has to be balanced by representing what is already learned vs. learned in the program.
- How is this represented in the theoretical models³¹?

Other closely related questions

- Human professional decision-making and “Noise” (Khaneman).
- Validation of asserted models: Range and uncertainty quantification of these models.
- Related regulatory history such as FDA interest *in silico* trials and FAA work toward model based certification.
- Historical Design of Experiments (DOE).
- Learning by transfer.

Discussion and next steps

- What shall we do prior to, during, and after IS2023?
- Outreach to Oli de Weck.
- Planning Primer on STEM Foundations Impacts on Engineering
-
-
-
-

References

1. de Weck, O., “The First Law of Systems Science: Conservation of Complexity”, INCOSE IW2023, Los Angeles, CA, January 2023.
2. de Weck, O, et al, “A Posteriori Design Change Analysis for Complex Engineering Projects”, *ASME J. of Mechanical Design*, Oct. 2011, Vol. 133.
3. Sinha, K., “Structural Complexity and its Implications for Design of Cyber--Physical Systems”, Thesis, MIT, 2014.
4. Sinha, K, and de Weck, O., “Matrix Energy as a Measure of Topological Complexity of a Graph”.
5. Schindel, W. “SE Foundation Elements: Discussion Inputs to INCOSE Vision 2035 Theoretical Foundations Section”, INCOSE Patterns Working Group, May, 2020.
6. ----- “Got Phenomena? Science-Based Disciplines for Emerging Systems Challenges”, in *Proc of INCOSE International Symposium*, 26(1), 2256–2271. <https://doi.org/10.1002/j.2334-5837.2016.00293>, 2016.
7. Schindel, W., and Dove, R., “Introduction to the ASELCM Pattern”, *Proc. of INCOSE IS2016*, Edinburg, UK, 2016
8. Marsden, J., et al, *Introduction to Mechanics and Symmetry*, 2nd ed., Springer, 1999.
9. Rojo, A., and Bloch, A. (2018). *The Principle of Least Action: History and Physics*, Cambridge U Press.
10. Lanczos, C. (1970). *The Variational Principles of Mechanics*, U. of Toronto Press, Fourth Edition.
11. Morin, D. (2007). *Introduction to Classical Mechanics*, Cambridge U Press, 2007
12. Capobianco, S., and Toffoli, T., “Conserved quantities in discrete dynamics: what can be recovered from Noether’s theorem, how, and why?” *Nat Comput* (2012) 11:565–577 DOI 10.1007/s11047-012-9336-7 Springer 2012.
13. Greydanus, S. et al, “Hamiltonian Neural Networks”, *Proc. of Conference on Neural Information Processing Systems (NeurIPS 2019)*, Vancouver, Canada. 2019.
14. Bertalan, T., et al, “On Learning Hamiltonian Systems from Data”, *Chaos* 29, 121107 (2019); <https://doi.org/10.1063/1.5128231>, 2019.
15. Kalman, R. E., “A New Approach to Linear Filtering and Prediction Problems”, *Transactions of the ASME–Journal of Basic Engineering*, 82 (Series D): 35-45, 1960.
16. Khaneman, D., et al, *NOISE: A Flaw in Human Judgement*.
17. Valerdi, R., Boehm, B., Rieff, J., “COSYSMO: A Constructive Systems Engineering Cost Model Coming of Age”, in *Proc of INCOSE IS2003*.
18. Valerdi, R., Wheaton, M., Rief, J., Wang, “Lessons Learned from Industrial Validation of COSYSMO”, in *Proc. of INCOSE IS2007*.
19. Wang, G, Pena, M., Roedler, G., Valerdi, R., “A Generalized Systems Engineering Reuse Framework and Its Cost Estimating Relationship”, in *Proc of INCOSE IS2014*.